Example Sheet 9

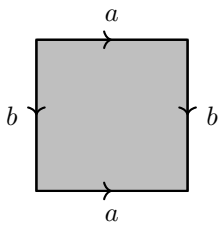
Please let me (Martin) know if any of the problems are unclear or have typos. Please turn in a solution to one of the three Exercises by 14:00 on 6/12/2018, to the dropoff box in front of the undergraduate office. If you collaborate with other students, please include their names.

(9.1) Show that a CW complex X is path-connected if and only if it is connected.

Hint You can show this either directly, or using the fact from the lecture that a CW complex is locally contractible.

(9.2) List all surjective homomorphisms from $\mathbb{F}_2 = \mathbb{Z} * \mathbb{Z}$, the free group of rank two, to $\mathbb{Z}_2$, the finite group with two elements. Prove your list is complete.

(9.3) Use the Seifert-van Kampen Theorem to compute the fundamental group of the torus $\mathbb{T}^2$, by using the following model of the torus:



You may use the fact, shown in an earlier exercise, that the torus with a missing point deformation retracts to the figure-eight $S^1 \vee S^1$, and that the fundamental group of this is $\mathbb{Z} * \mathbb{Z}$.